NAME (underline family name):

STUDENT NUMBER: **SIGNATURE:**

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATH 263

ORDINARY DIFFERENTIAL EQUATIONS AND LINEAR ALGEBRA

Examiner: G. Schmidt **Date:** Tuesday, April 18, 2006
Associate Examiner: D. Kelome **Time:** 9:00 AM - 12:00 AM

Instructions

1. Write your name and student number on this examination script.
2. No books, calculators or notes allowed.
3. This examination booklet consists of this cover, 9 pages of questions and 2 blank pages (12 numbered pages in all). Please take a couple of minutes in the beginning of the examination to scan the problems. (Please inform the invigilator if the booklet is defective.)
4. Answer all questions. You are expected to show all your work. All solutions are to be written on the page where the question is printed. You may continue your solutions on the facing page. If that space is exhausted you may continue on the blank pages at the end, clearly indicating any continuation on the page where the question is printed.
5. Your answers may contain expressions that cannot be computed without a calculator.
6. Circle your answers where confusion could arise.

GOOD LUCK!

Score Table

Problem	Points	Out of
1.		10
2.		10
3.		10
4.		10
5.		12
6.		12
7.		14
8.		14
9.		8
Total:		100

1. (10 marks) Find the solution $y(x)$ of

$$xy' + y = \ln x, \ y(1) = 3.$$

2. (10 marks) Using the substitution $v = y/x$, find the solution $y(x)$ of

$$xyy' = x^2 + y^2,\ y(1) = 2.$$

3. (10 marks) Solve implicitly

$$xdx + (x^2y + 4y)dy = 0,\ y(4) = 0.$$

4. (10 marks) Find the solution $y(x)$ of

$$y'' + y = x\cos x - \cos x \quad y(0) = 2.\ y'(0) = 1.$$

5. (12 marks) Find the general solution $y(x)$ of

$$y'' + 3y' + 2y = \frac{1}{1+e^x}.$$

6. (12 marks) Use Laplace transforms, and the table which follows, to solve

$$y'' + y = f(t),\ y(0) = 0, y'(0) = 2,$$

where $f(t) = 2$ for $0 \le t \le \pi$ and 1 for $t > \pi$.

function $f(t)$	Laplace transform $F(s)$
1	$1/s \quad (s > 0)$
t^n	$n!/s^{n+1} \quad (s > 0)$
e^{at}	$1/(s-a) \quad (s > a)$
$\sin at$	$a/(s^2 + a^2) \quad (s > 0)$
$\cos at$	$s/(s^2 + a^2) \quad (s > 0)$
$e^{-at} f(t)$	$F(s + a)$
$H(t-a)$ or $u_a(t)$ $(a \ge 0)$	$e^{-as}/s \quad (s > 0)$
$\delta(t-a) \quad (a > 0)$	e^{-as}
$H(t-a)f(t-a)$ or $u_a(t)f(t-a)$	$e^{-as}F(s)$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) \ldots - f^{(n-1)}(0)$
$f * g(t) = \int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$

7. (14 marks in total) Let

$$A = \begin{pmatrix} 0 & -1 & -1 \\ -2 & -1 & -2 \\ 1 & 1 & 2 \end{pmatrix}.$$

(a) (4 marks) Given that 1 is an eigenvalue find a basis for the corresponding eigenspace.
(b) (4 marks) Find the other eigenvalue(s) and bases for their eigenspace(s).
(c) (4 marks) Find orthogonal Q and diagonal D such that $A = QDQ^T$.
(d) (2 marks) Give a geometric interpretation of the transformation from $\mathbf{R}^3$ to itself defined by the matrix.

8. (14 marks in total) Consider the matrix $A = \begin{pmatrix} 0 & 2 \\ 1 & 1 \end{pmatrix}$.

(a) (6 marks) Diagonalize A.
(b) (6 marks) Evaluate e^{At}, for t real.
(c) (2 marks) Using e^{At}, write down an expression for the solution of the system of the system of differential equations

$$\begin{aligned} x_1' &= 2x_2, & x_1(0) = a, \\ x_2' &= x_1 + x_2, & x_2(0) = b. \end{aligned}$$

9. (8 marks in total) Let A be a real 3x3 matrix.
(a) (2 marks) Given that -2 and $3+2i$ are eigenvalues corresponding respectively to eigenvectors $(1,1,1)$ and $(1-3i,i,2)$ respectively write down the third eigenvalue and its corresponding eigenvector.
(b) (4 marks) Write down a basis of real solutions of the system $\dfrac{d\mathbf{x}}{dt} = A\mathbf{x}$.
(c) (2 marks) For which initial values $\mathbf{x}(0)$ does the corresponding solution $\mathbf{x}(t)$ satisfy $\mathbf{x}(t) \longrightarrow 0$ as $t \longrightarrow -\infty$?

ONTINUATION PAGE FOR QUESTION NUMBER ☐

You must refer to this continuation page on the page where the question is printed.

ONTINUATION PAGE FOR QUESTION NUMBER ☐

You must refer to this continuation page on the page where the question is printed.